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DESIGN OF SUBMERGED TURBULENT JETS OF GASES OF DIFFERENT DENSITIES

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We present results of a theoretical and experimental investigation of submerged gas jets in the range of density variations ($\rho_j/\rho_e = 0.05$-10).

A large number of studies [1-9] have been devoted to an investigation of the features of the propagation of submerged jets. Below, an attempt is made to generalize the available experimental data [2, 5, 8] for jets of various densities and to calculate certain characteristics of such jets.

For construction of the graphs (Fig. 1) we assumed the jets to be a point source placed at the pole with initial momentum equal to k_j ($k_j = \rho_j u_j^2 F_j$). The distance to the pole x_p was found from the construction of the profiles ρu^2, u, and ΔT or c at various distances from the nozzle at the $x_1 - r$ plane following the drawing of the straight lines passing through the points at which the velocity head, the velocity, and the excess temperature or concentration at each cross section of the jet attained half of their maximum (on the jet axis) value, i.e., we constructed the straight lines $r^q_{0.5}$, $r^u_{0.5}$, and $r^T_{0.5}$, originating from a single point – the pole [5].

From Fig. 1 we see that the width of the profiles ρu^2, u, and ΔT increase with decreasing density of the jet and dimensionless profiles of the excess temperatures $\Delta T/\Delta T_m$ and the concentrations c/c_m coincide [5]. In this case with an accuracy that is acceptable in practice the velocity distribution over transverse cross sections of the indicated jets is described by the theoretical profile

$$\frac{u}{u_m} = \left[1 - \left(0.44\,\frac{r/x_1}{r^u_{0.5}/x_1}\right)^{3/2}\right]^2, \tag{1}$$

and the distribution of the excess temperatures or concentrations is described by a profile which can be written as

$$\frac{\Delta T}{\Delta T_m} = \frac{c}{c_m} = \left[1 - \left(0.44\,\frac{r/x_1}{r^T_{0.5}/x_1}\right)^{3/2}\right]^2. \tag{2}$$

In Eqs. (1) and (2) the ratios of the half-maximum values of the transverse coordinates, the velocity and the temperature, $r^u_{0.5}$ and $r^T_{0.5}$ to their limiting values r^u_{lim} and r^T_{lim} are the same for all jets:

$$\frac{r^u_{0.5}}{r^u_{\text{lim}}} = 0.44, \quad \frac{r^T_{0.5}}{r^T_{\text{lim}}} = 0.44. \tag{3}$$

The change in the coefficient of the half-width of the jet with respect to the velocity $C^u_{0.5} = r^u_{0.5}/x_1$ and the temperature $C^T_{0.5} = r^T_{0.5}/x_1$ as a function of the relative density of the jet ρ_j/ρ_e are shown in Fig. 2. In this figure we show the variation of the coefficient of the half-width of the jet with respect to the velocity head $C^q_{0.5} = r^q_{0.5}/x_1$.

The distribution of $\rho u^2/\rho_m u_m^2$ in cross sections of the isothermal jet of air $\rho_j/\rho_e = 1.0$ (Fig. 1) corresponds to the theoretical profile obtained from Eq. (1):

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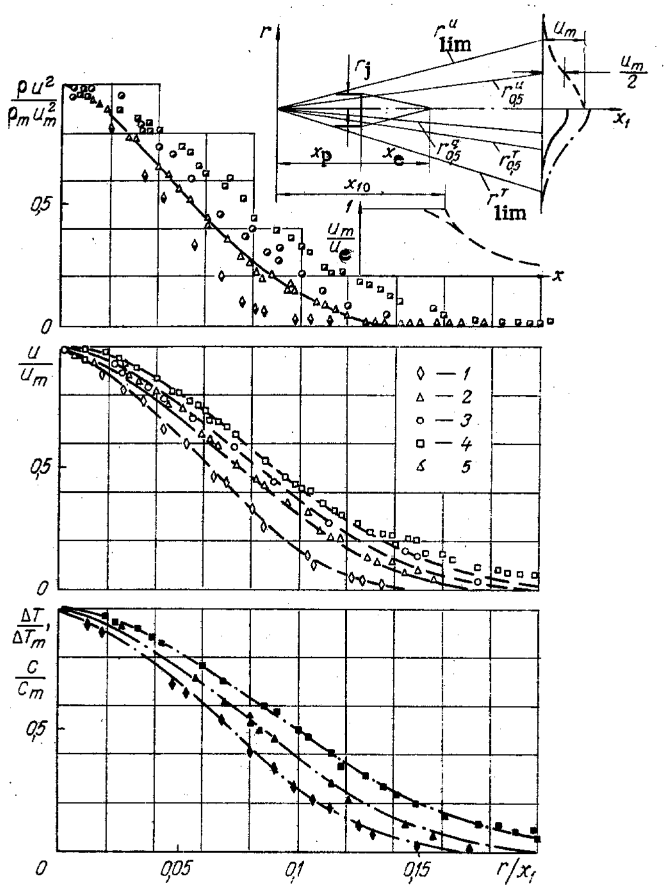

Fig. 1. Profiles of velocity heads $\rho u^2/\rho_m u_m^2$ (points, half shaded), velocities u/u_m (open figures) and temperatures $\Delta T/\Delta T_m$ or c/c_m (filled figures) for jets of various densities [solid curve) $\rho u^2/\rho_m u_m^2$; dashed curve) u/u_m; dot–dash curve) $\Delta T/\Delta T_m$ – prediction]. Here and below, see Table 1 for the notation of the experimental points.

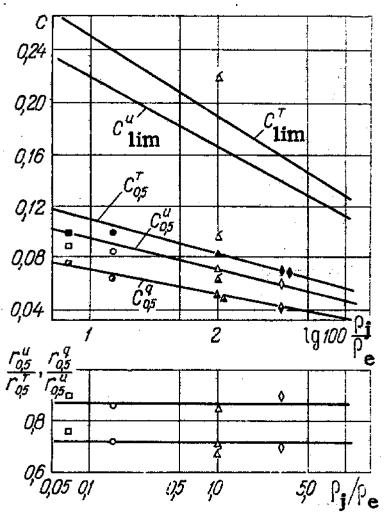

Fig. 2. Variation of expansion coefficient of the half-jet ($C^q_{0.5}$, $C^u_{0.5}$, and $C^T_{0.5}$) and the jet ($C^u_{\lim}$; $C^T_{\lim}$) and of the ratios $r^q_{0.5}/r^u_{0.5}$ and $r^u_{0.5}/r^T_{0.5}$ as a function of the density of the jet (lines – prediction).

TABLE 1. Process Parameters of the Jets

Working medium of jet	r_j, mm	x, mm	$\frac{\rho_j}{\rho_e}$	Experimental points	According to data of
Freon	2,5	70–145	3,2	1	[5]
Air	5,0	50–150	1,0	2	[2]
Helium	2,5	70–145	0,125	3	[5]
Air	15	250–500	0,067	4	[8]
Air	45	600–1600	1,0	5	Expt. of Trüpel [1]

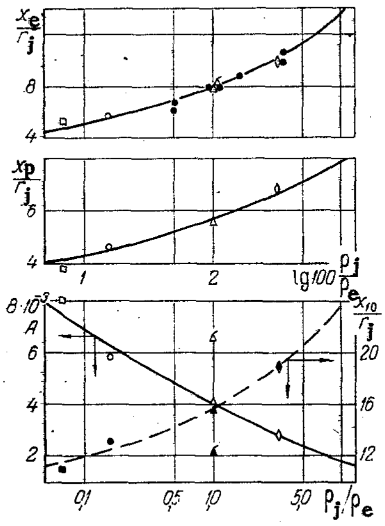

Fig. 3. Dependence of length of initial section x_e/r_j, distance up to the pole of the jet x_p/r_j, the interval A, and the quantities x_{10}/r_j on the jet density [black circles) – data of the work of Kukes; solid curves) – calculation].

$$\frac{\rho u^2}{\rho_m u_m^2} = \left(\frac{u}{u_m}\right)^2. \tag{4}$$

We should note that the ratio of the transverse coordinates of the jet, where the velocity head equals half of the maximum value, to the coordinates with the half-value velocity $r^q_{0.5}/r^u_{0.5}$ for the jets being considered does not change and equals 0.71 (Fig. 2).

The turbulent Prandtl numbers in cross sections of the indicated jets also are independent of the density and equal 0.87 (Fig. 2). These numbers were determined as:

$$\mathrm{Pr_T} = r^u_{0.5}/r^T_{0.5}. \tag{5}$$

The determination of the exterior boundary of the jet based on the experimental data is connected with an entire series of difficulties and leads to large errors. Taking this last fact into account, the exterior boundary of the jet based on velocity $r^u_{\lim}$ and temperature $r^T_{\lim}$ was determined according to the corresponding half-width from Eqs. (3).

On the basis of the data obtained in this way we found the expansion coefficients of the fundamental sections of the jet according to velocity

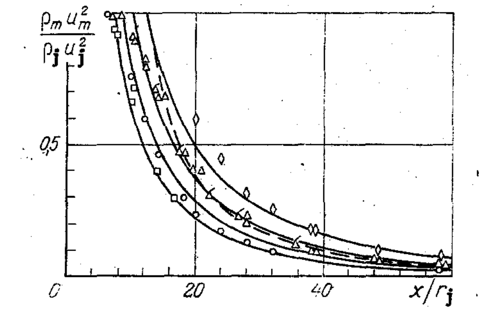

Fig. 4. Variation of velocity head on the axis along the length for jets of various densities (curves – prediction).

$$C = C^u_{\mathrm{lim}} = \frac{r^u_{\mathrm{lim}}}{x_1} = \frac{r^u_{0.5}/x_1}{0.44} = \frac{C^u_{0.5}}{0.44} \tag{6}$$

and according to temperature

$$C^T_{\mathrm{lim}} = \frac{r^T_{\mathrm{lim}}}{x_1} = \frac{r^T_{0.5}/x_1}{0.44} = \frac{C^T_{0.5}}{0.44}\,. \tag{7}$$

In order to determine the expansion coefficient of the jet $C = C^u_{\mathrm{lim}}$ as a function of its density we can recommend the following empirical formula:

$$C = C^u_{\mathrm{lim}} = 0.273 - 0.053\,\lg 100\frac{\rho_j}{\rho_e}\,. \tag{8}$$

From Fig. 2 we see that for a jet of an incompressible fluid ($\rho_j/\rho_e = 1$) the expansion coefficient of the jet C equals 0.17, which differs considerably from the value that was assumed earlier – which was equal to 0.22 [1].

If we assume that for $\rho_j/\rho_e = 1$ the value of the coefficient C = 0.22 is correct [1], then from (6) it follows that

$$C^u_{0.5} = C\cdot 0.44 = 0.22\cdot 0.44 = 0.097.$$

Based on the experimental data of [8] we see (Fig. 2) that even for a strongly heated air jet ($\rho_j/\rho_e = 0.067$) we have a somewhat lower value of the coefficient $C^u_{0.5} = 0.09$, and for a jet of an incompressible fluid, according to Fig. 2, $C^u_{0.5} = 0.073$ and, hence, $C = C^u_{\mathrm{lim}} = 0.073/0.44 = 0.166$.

Based on the experimental data that have been obtained and also the data in the literature we constructed graphs of the variation of the length of the initial section of the jet x_e/r_j and the distance from the section of the jet up to the pole of the jet x_p/r_j as a function of the density of the jet ρ_j/ρ_e (Fig. 3). From Fig. 3 we see that with increasing jet density there is an increase in the distance upto the pole owing to the decrease in the width of the jet and the length of the initial section – as a result of the reduction of the apparent mass along its length.

The length of the initial section x_e/r_j and the distance from the section of the jet up to the pole x_p/r_j as a function of the density of the jet can be determined from the following empirical formulas:

$$\frac{x_e}{r_j} = 1.325\left(\lg 100\frac{\rho_j}{\rho_e}\right)^2 + 1.161\,\lg 100\frac{\rho_j}{\rho_e} + 5, \tag{9}$$

$$\frac{x_p}{r_j} = 0.41\left(\lg 100\frac{\rho_j}{\rho_e}\right)^2 + 0.133\,\lg 100\frac{\rho_j}{\rho_e} + 3.72. \tag{10}$$

If we write the equation of conservation of momentum on the section of the jet and in an arbitrary cross section of the jet

$$\rho_j u_j^2 F_j = \int_0^F \rho u^2 dF \tag{11}$$

and we assume the jet to be a point source situated at the pole (Fig. 1), then Eq. (11) after simple transformations can be represented as

$$\frac{\rho_m u_m^2}{\rho_j u_j^2} = \frac{1}{\left(\frac{x_1}{r_j}\right)^2 2 \int_0^{r^u_{\lim}/x_1} \frac{\rho u^2}{\rho_m u_m^2} \frac{r}{x_1} \frac{dr}{x_1}} = \frac{1}{\left(\frac{x_1}{r_j}\right)^2 A}. \tag{12}$$

From Fig. 1 it follows that the dimensionless profiles of the velocity head $\rho u^2/\rho_m u_m^2$ with an accuracy acceptable in practice can be assumed to be universal along the length of the jet. Then the value of the integral A will not vary along the length of the jet and will depend only on the density of the jet (Fig. 3) and for $\rho_j/\rho_e = 1$ it will have a value $A = 4.1 \cdot 10^{-3}$.

However, based on the experimental data of Trüpel ($\rho_j/\rho_e = 1$) the quantity A has a considerably greater value and is equal to $6.25 \cdot 10^{-3}$.

Based on the values found for the integral A as a function of the density of the jet, on the basis of (12) we determined the distances (in calibers of the jet x_{10}/r_j) from the pole to the cross section of the jet, in which the initial velocity head ($\rho_m u_m^2/\rho_j u_j^2 = 1$) is preserved. The quantity x_{10}/r_j according to the scheme of the jet (Fig. 1) is equal to

$$\frac{x_{10}}{r_j} = \frac{x_e}{r_j} + \frac{x_p}{r_j} + \frac{x_{tr}}{r_j}. \tag{13}$$

From a comparison of the calculated curve for x_{10}/r_j and the experimental curves for x_e/r_j and x_p/r_j it follows that the relative length of the transitional section x_{tr}/r_c for the jets being investigated is equal to 2.

Since according to the experimental data of Trüpel [1] the pole is situated on the cross section of the jet, and the value of the integral $A = 6.52 \cdot 10^{-3}$, then the length of the initial section of the jet, calculated from Eq. (12), equals 12.4. The result obtained was found to be in obvious disagreement with numerous experimental data, including that of the experiments of Trüpel (Fig. 4).

Equation (12) enables us to determine the variation of the velocity head along the jet axis $\rho_m u_m^2/\rho_j u_j^2$.

In Fig. 4 we compare the calculated and experimental data based on the variation of $\rho_m u_m^2/\rho_j u_j^2$ for jets of different densities. It is seen that they are in good agreement. However the fall in the velocity head along the length of the jet, calculated for the value of the integral $A = 6.52 \cdot 10^{-3}$ with respect to (12) occurs considerably more intensely and disagrees with the available experimental data (dashed line).

As a result of the generalization of the experimental data based on the variation of the coefficient of the half-width of the jet with respect to the velocity $C^u_{0.5} = r^u_{0.5}/x_1$ as a function of its density, and also on the basis of the equation of conservation of momentum, it follows that we should assume it to be established that the expansion coefficient of the jet C of an incompressible fluid ($\rho_j/\rho_e = 1$) equals 0.17, and not 0.22, as was assumed earlier.

NOTATION

r_j	is the jet radius;
x_1	is the distance from the pole along the axis of the jet;
u_j,	is the jet flow rate at the exit from the jet;
ρ_j, ρ_e	are the density of the jet and of the environment;
$C^q_{0.5}$, $C^u_{0.5}$, $C^T_{0.5}$	are the expansion coefficients of the jet, where the velocity head, the velocity, and the temperature are equal to their half-maximum values;
$C^u_{\lim}$, $C^T_{\lim}$	are the expansion coefficients of the jet with respect to velocity and temperature.

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LAMINAR WAVE FLOW OF A FILM OF A VISCOPLASTIC LIQUID

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We solve the problem of a laminar wave falling down a vertical surface for a thin film of a viscoplastic Shvedov–Bingham liquid.

Films of liquid falling down a vertical surface have a wave nature for flow rates exceeding a certain critical value. According to the available experimental data, the increase in the coefficients of heat and mass transfer, due to the wave formation, can reach 50% and greater. Such a type of flow is rather frequently encountered in various applications, in particular, in processes and apparatus of chemical technology. Flowing media, processable in such technological processes, e.g., viscoplastic liquids, are often rheologically complex.

We consider laminar wave flow of a thin film of a viscoplastic liquid falling down a vertical surface, which satisfies the rheological Shvedov–Bingham law

$$\tau' = \tau_0 + \mu \frac{\partial u'}{\partial y'}, \quad |\tau'| > \tau_0, \qquad \frac{\partial u'}{\partial y'} = 0, \quad |\tau'| \leqslant \tau_0. \tag{1}$$

Here τ_0 is the yield point and μ is the plastic viscosity.

We consider the case of long waves, i.e., waves whose length is great in comparison with the thickness of the film. We introduce the dimensionless variables and parameters

$$lt = \alpha U t', \ lx = \alpha x', \ ly = y', \ Uu = u', \ \alpha U v = v', \quad lh = h', \ \rho U^2 p = p', \tag{2}$$

$$\mathrm{Re} = Ul\frac{\rho}{\mu}, \ \mathrm{Fr} = \frac{U^2}{gl}, \ W = lU^2\frac{\rho}{\sigma}, \ S = \frac{\tau_0 l}{\mu U}, \tag{3}$$

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